

Exercise 30

If $f(t) = \sec t$, find $f''(\pi/4)$.

Solution

Calculate the first derivative.

$$\begin{aligned}f'(t) &= \frac{d}{dt}(\sec t) \\ &= \sec t \tan t\end{aligned}$$

Calculate the second derivative using the product rule.

$$\begin{aligned}f''(t) &= \frac{d}{dt}[f'(t)] \\ &= \frac{d}{dt}(\sec t \tan t) \\ &= \left[\frac{d}{dt}(\sec t) \right] \tan t + \sec t \left[\frac{d}{dt}(\tan t) \right] \\ &= (\sec t \tan t) \tan t + \sec t (\sec^2 t) \\ &= \sec t \tan^2 t + \sec^3 t\end{aligned}$$

Evaluate the second derivative at $t = \pi/4$.

$$f''(\pi/4) = \sec(\pi/4) \tan^2(\pi/4) + \sec^3(\pi/4) = (\sqrt{2})(1)^2 + (\sqrt{2})^3 = 3\sqrt{2}$$